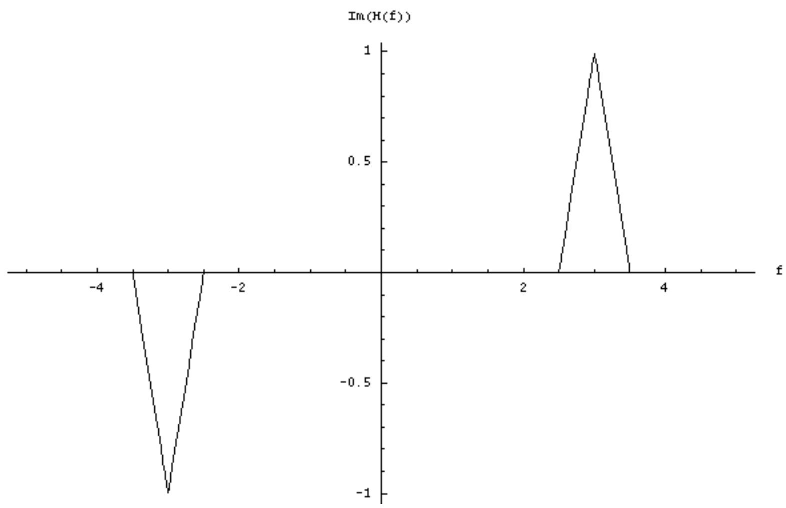
ECE 343: Signals and Systems

Test 2: Fourier Transforms and Fourier Series

1. Let $x(t) = \Pi(\frac{t}{\tau}) \cos^2(\pi \frac{t}{\tau}), \quad \tau > 0$. This is called a raised-cosine pulse.
   (a) Sketch $x(t)$.
   (b) Find the Fourier transform $X(f)$ of $x(t)$.
   (c) Compute the integral $\int_{-\infty}^{\infty} X(f)df$.

2. Let $x(t) = 2\cos^2(\pi t) + j\sin(2\pi t)$.
   (a) Find the (smallest possible) period of $x(t)$.
   (b) Find the Fourier series coefficients $X[k]$, for all k.
   (c) Find the power of $x(t)$.

3. A bandpass filter has the transfer function shown in the figure (*Warning: The plot is of the imaginary part of the transfer function. The real part is zero.*). The filter is an LTI system.



   (a) Find the impulse response of the system.
   (b) The filter is given an input of $\cos(2\pi \frac{13}{4} t)$. What is the output?

4. Evaluate the integral

$$\int_{-\infty}^{\infty} \mathrm{sinc}^2(t-\lambda)\mathrm{sinc}(\lambda)d\lambda$$

Some useful transforms:

$$\begin{aligned}
\mathcal{F}\{\delta(t)\} &= 1 \\
\mathcal{F}\{\Pi(t)\} &= \mathrm{sinc}(f) \\
\mathcal{F}\{\Lambda(t)\} &= \mathrm{sinc}^2(f) \\
\mathcal{F}\{e^{-t}u(t)\} &= \frac{1}{1+2\pi jf} \\
\mathcal{F}\{u(t)\} &= \frac{1}{\pi jf}
\end{aligned}$$